A Study on MRP with Using Leads Time, Order Quality and Service Level over a Single Inventory

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Abstract
This study intends to examine Material Requirement Planning (MRP) software parameterization under uncertainties. The actual lead time can be considered as a random variable as has random deviations. In this study, Material Requirement Planning is considered regarding policy of Periodic Order Quantity (POQ). Further, this study aims to search optimal MRP time phasing in compliance with periodicity of the POQ policy. Indeed, studying MRP parameters using random replenishment leads time, POQ policy and service level constraint over a single inventory is a leading issue in supply planning with respect to MRP approach, and the reason for this lies in a fact that inappropriate planned lead times under lead time uncertainties result in large and costly inventories or inadequate customer service levels. The algorithms used in this study minimize the sum of the setup and holding costs beside meet the constraint on the service level. The approach applied in this study does not need to apply normal probability distributions, yet mainly aims to find a form very close to objective function, that is valid for any regarding actual lead times.

Keywords: Material Requirement Planning (MRP); Periodic Order Quantity (POQ); service level constraint

Introduction
Crucial problem in supply planning is a leading problem, that overstocking or stockout situations emerge in case of existing inadequate inventory control policy. The generated inventories in overstocking situation are expensive, but shortages and penalties due to unsatisfied customer demands emerge in case of stockout situations. A conventional accepted approach for replenishment planning in major companies has been regarded the very Material Requirements Planning (MRP), where on MRP-based software tools are further accepted (Wee HM, Shum YS. (1999)). The practical aspect of MRP orients regarding that it is based on comprehensible rules, providing cognitive support as well as a powerful information system for decision making. MRP orients regarding the presupposition that both demand and lead time are deterministic, yet most production systems are stochastic, e.g. variability of actual supplier load can be regarded to define a random lead time considering that when a supplier furnishes several clients, its load depends on the timing of all client orders, if total demand outstrips production capacity, the lead time increases (Browne J., Harben J. and Shivnan J. (1996)). Increasing randomness of lead times relies on many external factors including outsourced production overseas can introduce some randomness via shipping perturbations, the orders might not arrive by the due date because of work stoppage or delays attributable to the weather. Machine breakdowns, absenteeism, other random variations of capacity are from random factors and unpredictable events that cause deviations in actual lead times from planning times. The deterministic assumptions of MRP can be often too restrictive as aforementioned. Lead time is a principal factor foreseeing production and lead time randomness affects seriously ordering policies, inventory levels and customer service levels as shown in previous studies. The MRP approach can be outlined to uncertainties by
searching optimal values for its parameters. Choosing these parameters in sufficiency increases the effectiveness of MRP techniques. MRP parameterization is one of the essential issues for companies in industrial situations, that is, MRP offsetting under uncertainties.

Several MRP parameters include planned lead time, safety stock, lot-sizing rule, freezing horizon, planning horizon, etc. concerning safety stock calculation for random demand of finished products, there exist extensive publications. Certain parameters seem not to be sufficiently examined, e.g. planned lead time, optimal parameterization is also an open issue.

The planned lead time can contain safety lead time if actual lead time is random, that is, the sum of the forecasted and safety lead times is considered as planned lead time. The search for optimal value of safety lead time, and, consequently, for planned lead time, is a crucial issue in supply planning with the MRP approach. The problem of planned lead times optimization in case of using safety a lead time has been given scant attention in the literature. Mostly, average values of probability distributions of actual lead times are used. A longer than necessary planned lead time creates excessive work in progress.

An overview on previous studies

MRP has been a very popular and widely used multilevel inventory control method since 1970s. The application of this popular tool in materials management has greatly reduced inventory levels and improved productivity (Wee and Shum, 1999). The introduced MRP was the first version of MRP system, named as Materials Requirements Planning (MRP I). Later, several MRP systems were extended into other versions including Manufacturing Resources Planning (MRP II) and Enterprise Resources Planning (ERP) (Browne et al., 1996). Van Donselaar and Gubbels (2002) compare MRP and line requirements planning (LRP) for planning orders. Their research basically focuses on minimizing the system inventory and system nervousness. They also discuss and propose LRP technique to achieve their goals. Minifie and Davies (1990) developed a dynamic MRP controlled manufacturing system simulation model to study the interaction effects of demand and supply uncertainties. These uncertainties were modelled in terms of changes in lot size, timing, planned orders and policy fence on several system performance measures, namely late deliveries, number of setups, ending inventory levels, component shortages and number of exception reports. Billington et al. (1983) suggest a mathematical programming approach for scheduling capacity constrained MRP systems. They propose a discretetime, mixed integer linear programming formulation. In order to reduce the number of variables, and thus the problem size, they introduce the idea of product structure compression (Billington PJ, McClain JO, Thomas J. (1983)).

Kumar (1989) studies a single period model (one assembly batch) for a multi-component assembly system with stochastic component lead times and a fixed assembly due date and quantity. The problem is to determine the timing of each component order so that the total cost composed of the component holding and product tardiness costs is minimized (Kumar, A. (1989)). Chauhan et al. (2009), presents an interesting single period model. Their approach considers a punctual fixed demand for one finished product. Multiple types of components are needed to assembly this product. The objective is to determine the ordering time for each component such as to minimize the sum of expected holding and backlogging costs (Chauhan, S.S., Dolgui, A., Proth, J.M. (2009)).

Kanet and Sridharan (1998) examined late delivery of raw materials, variations in process lead times, interoperation move times and queue waiting times in MRP controlled manufacturing environment. To model such environment, they represented demand by inter arrival time rather than defined from the master production schedule (Kanet JJ, Sridharan SV. (1998)).

Wilhelm and Som (1998) present an inventory control approach for an assembly system with several types of components. Their model focuses on a single finished product inventory, so the interdependence between inventory levels of different components is once again neglected (Wilhelm, W.E., Som, P. (1998)).

Axsäter (2005) considers a multi level assembly system where operation times are independent random variables. The objective is to choose starting times (release dates) for different operations in order to minimize the sum of the expected holding and backlogging costs (Axsäter, S. (2005)).
The paper (Louly and Dolgui, 2002) considers the case of the objective function minimizing the sum of average holding and backlogging costs. While Louly et al. (2008) study the case when backlogging cost is replaced with a service level constraint. The obtained models for optimal planned lead times represent generalizations of the discrete Newsboy model (Louly, M.A., Dolgui, A. (2002)).

Yenisey (2006) applied a flow network model and solved a linear programming method for MRP problems that minimized the total cost of the MRP system. Mula et al. (2006) provided a new linear programming model for medium term production planning in a capacity constrained MRP with a multiproduct, multilevel, and multi period production system. Their proposed model comprised three fuzzy sub models with flexibility in the objective function, market demand, and capacity of resources (Mula, J., Poler, R., and Garcia, G. P. 2006).

**Optimization problem**

**Inventory model**

The following model will be considered in this paper regarding the particularities of MRP parameterization: Items are ordered from external suppliers to satisfy the customer demand, Probability distribution of procurement time L is known, the customer demand is known and is constant for all time buckets and equal to D. further, unit holding cost h per period, and setup cost c are also known. POQ policy is used where on items are ordered at every p periods. The orders for items are made at the beginning of the periods kp+1, k=0,1,2,....., and there does not exist any order in the periods kp+r, r=2,3,.....,p. thereby, the supply orders Q are constant Q=Dp. The product demands are satisfied at the end of each period and unsatisfied demands are backordered that have to be satisfied during the subsequent time buckets. This model aims to search the optimal values for parameters p and x, mentioned that desired service level, unit holding cost, setup cost, i.e. the cost of a supply order, upper value of lead time distribution, lead time of items ordered at the beginning of the period k, demand per period, periodicity of replenishments, supply order quantity, planned lead time, max(z,o) used to shown 1-r, h, c, u, L, D, p, Q, x and Z.

Planned lead time can be greater than the forecasted lead time as the lead time is a random variable, and the difference mentioned the safety lead time. Demand D is considered constant and the quantities considered the same and equal to Dp so that the optimal planned lead time x is the same for all orders, that it provides the initial inventory Dx. In this study, an approach is proposed to optimize the planned lead time x and periodicity p of POQ policy minimizing the sum of setup and holding costs regarding a service level constraint, resulted that the actual lead times are generally random.

**Analyze the criterion**

Proofs are made here to acquire an analytical expression for the criterion as a function of decision variables. According to the fact that the maximal value for lead time is equal to u, the orders made in the earlier u-1 periods may not have achieved. This can be used to obtain the distribution of probability for the number \(N_{p,m}\) of expected deliveries at the end of the period \(m=kip+r\). let \(L^{m+1}, J=r, r+p, r+2p, ......., r+((u-1-p)/p)p\) be the lead time of the orders made at the beginning of the periods kp+1, (k-1)p+1,....., (k-((u-1-p)/p))p+1. If \(L^{m+1}\) \(> j\), the order made in the period \(m+1-j\) is delivered after the end of the period m. let \(E\) be the binary function equal to 1 when the expression E is true, otherwise it is equal to 0. so, if \(L^{m+1}\) \(> j\) is equal to 1, then the order made at the period \(m+1-j\) is delivered after the end of the period m. thus, the random variables \(N_{p,m}\) can be represented as follows:

\[
N_{p,m} = N_{p,kip+r} = \sum_{j=0}^{(u-1-r)/p} 1_{q_k-jp+1 > j} \quad k \geq 0, \ p \in \{1,2,..,u-1\}, \\
r \in \{1,2,..,p\}
\]

(1)

The variables \(N_{p,m}\) are independent from decision variable x, therefore, such variables can be used to derive closed forms for the cost function.

Proposition: cost for the period \(m=kip+r, k \geq 0, p \in \{1,2,..,u-1\}, r \in \{1,2,..,p\}\) is equal to
Proof. To obtain an explicit expression for the cost function, it is sufficient to derive the expression for \(N_{p,m}^{kp+r}\). Indeed, the number of backlogged demands at the end of period \(m\) is equal to the cumulative required quantity \(D(kp+r)\) minus the initial item inventory \(Dx\) minus the delivered quantity \(D(pN_{p,kp+r}^+)\), while the inventory is its inverse. Therefore, the total satisfied demand is equal to \(D[pN_{p,kp+r} + N_{p,kp+r}^+ + r - p - x]\). Given that there is a setup cost \(c\) if \(r=1\), and using \(Z' = Z + Z\), the sum of the holding and setup costs at the end of period \(m\) can be written as

\[
C(x,p,N^{kp+r+}) = c \times 1 + Dh(x+p-r-pN^{kp+r}) + Dh(pN^{kp+r} + r - p - x)^+
\]

(2)

The cost of a single period \(kp+r\) is a random variable. To examine the multi-period problem, explicit closed forms should be obtained for the average cost and the average number of shortages on the infinite horizon, regarding the expressions below:

\[
C(X,p) = \lim_{m \to \infty} \frac{1}{m} \sum_{k=1}^{m} C(X,p,N^{kp,k})
\]

\[
S(X,p) = \lim_{m \to \infty} \frac{1}{m} \sum_{k=1}^{m} 1_{S(X,p,N^{kp,k}) > 0}
\]

(4)

Conclusion

To sum up, it was deduced Material requirement planning (MRP) is a plan for the production and purchase of the components used in making items in the master production schedule. It shows the quantities needed and when manufacturing intends to make or use them. MRP is a commonly accepted approach for replenishment planning in major companies. The MRP based software tools are accepted readily. Most industrial decision makers are familiar with their use. The practical aspect of MRP lies in the fact that this is based on comprehensible rules, and provides cognitive support, as well as a powerful information system for decision making. The model suggested in this paper can be used to better estimate these coefficients by using statistics on the procurement lead times for each supplier and taking into account the holding and setup costs. This is a multi-period model with no major restriction on the type of the lead time distribution. All discrete distributions can be used. The decision variables are integer; they represent the periodicity and planned lead time for items.

References